The focus of Mathematics II is on quadratic expressions, equations, and functions; comparing their characteristics and behavior to those of linear and exponential relationships from Mathematics I as organized into 6 critical areas, or units. The need for extending the set of rational numbers arises and real and complex numbers are introduced so that all quadratic equations can be solved. The link between probability and data is explored through conditional probability and counting methods, including their use in making and evaluating decisions. The study of similarity leads to an understanding of right triangle trigonometry and connects to quadratics through Pythagorean relationships. Circles, with their quadratic algebraic representations, round out the course. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Units	Includes Standards and Clusters	Mathematical Practices
Unit 1 Extending the Number System	•Extend the properties of exponents to rational exponents. •Use properties of rational and irrational numbers. •Perform arithmetic operations with complex numbers. •Perform arithmetic operations on polynomials.	
Unit 2 Quadratic Functions and Modeling	•Interpret functions that arise in applications in terms of a context. •Analyze functions using different representations. •Build a function that models a relationship between two quantities. •Build new functions from existing functions. •Construct and compare linear, quadratic, and exponential models and solve problems.	Make sense of problems and persevere in solving them. Reason abstractly and quantitatively.
Unit 3 Expressions and Equations	•Interpret the structure of expressions. •Write expressions in equivalent forms to solve problems. •Create equations that describe numbers or relationships. •Solve equations and inequalities in one variable. •Use complex numbers in polynomial identities and equations. •Solve systems of equations.	Construct viable arguments and critique the reasoning of others. Model with mathematics. Use appropriate tools strategically.
Unit 4 Applications of Probability	•Understand independence and conditional probability and use them to interpret data. •Use the rules of probability to compute probabilities of compound events in a uniform probability model. •Use probability to evaluate o	Attend to precision. Look for and make use of structure.
Unit 5 Similarity, Right Triangle Trigonometry, and Proof	•Understand similarity in terms of similarity transformations. •Prove geometric theorems. •Prove theorems involving similarity. •Use coordinates to prove simple geometric theorems algebraically. •Define trigonometric ratios and solve problems involving right triangles. •Prove and apply trigonometric identities.	Look for and express regularity in repeated reasoning.

Unit 6	•Understand and apply theorems about circles. •Find arc lengths and areas of sectors of circles. •Translate between the geometric description and the equation for a conic section. •Use coordinates to prove simple geometric theorem algebraically. •Explain volume formulas and use them to solve problems.		
*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit. †Note that solving equations follows a study of functions in this course. To examine equations before functions, this unit could come before Unit 2.			
Critical Area 1: Students extend the laws of exponents to rational exponents and explore distinctions between rational and irrational numbers by considering their decimal representations. In Unit 3, students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows x+1 = 0 to have a solution. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system.			
Unit 1 Extending the Number System			
Extend the properties of exponents to rational exponents.		N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5. N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.	
Use properties of rational and irrational numbers.Connect N.RN.3 to physical situations, e.g., finding the perimeter of a square of area 2.		N.RN.3 Explain why sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational.	
Perform arithmetic operations with complex numbers.Limit to multiplications that involve i2 as the highest power of i		N.CN.1 Know there is a complex number i such that i2 = −1, and every complex number has the form a + bi with a and b real. N.CN.2 Use the relation i2 = –1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	
Perform arithmetic operations on polynomials.Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x.		A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	Importantest wo M01. Problem Apply a specifie Apply relevant A-APR.1 — Und Apply a specifie Apply relevant

Unit 2 Quadratic Functions and Modeling		
Interpret functions that arise in appli-cations in terms of a context. Focus on quadratic functions; compare with linear and exponential functions studied in Mathematics I.	F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. ★F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	
Analyze functions using different rep-resentations.For F.IF.7b, compare and contrast absolute value, step and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range and usefulness when examining piecewise-defined functions. Note that this unit, and in particular in F.IF.8b, extends the work begun in Mathematics I on exponential functions with integer exponents. For F.IF.9, focus on expanding the types of functions considered to include, linear, exponential, and quadratic.Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.	F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★a.Graph linear and quadratic functions and show intercepts, maxima, and minima.b.Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. F.IF.8 Write a functiondefined by an expression in different but equivalent forms to reveal and explain different properties of the function. a.Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.★b.Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay. F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.	
Build a function that models a relation-ship between two quantities.Focus on situations that exhibit a quadratic or exponential relationship	F.BF.1 Write a function that describes a relationship between two quantities.★ a.Determine an explicit expression, a recursive process, or steps for calculation from a context.b.Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying expo-nential, and relate these functions to the model	

Build new functions from existing func-tions.For F.BF.3, focus on quadratic functions and consider including absolute value functions.. For F.BF.4a, focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as f(x) = x2, x>0.	F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.F.BF.4 Find inverse functions. a.Solve an equation of the form f(x) = c for a simple function fthat has an inverse and write an expression for the inverse. For example, f(x) = 2 x3 or f(x) = (x+1)/(x-1) for x ≠ 1	
Construct and compare linear, quadrat-ic, and exponential models and solve problems.Compare linear and exponential growth studied in Mathematics I to quadratic growth.	F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	
Unit 3 Expressions and Equations		
Interpret the structure of expressions.Focus on quadratic and exponential expressions. For A.SSE.1b, exponents are extended from the integer exponents found in Mathematics I to rational exponents focusing on those that represent square or cube roots.	A.SSE.1 Interpret expressions that represent a quantity in terms of its context.★a.Interpret parts of an expression, such as terms, factors, and coef-ficients.b.Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P.A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2).	
Write expressions in equivalent forms to solve problems.It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.	A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ a.Factor a quadratic expression to reveal the zeros of the function it defines.b.Complete the square in a quadratic expression to reveal the maxi-mum or minimum value of the function it defines.c.Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be re-written as (1.151/12)12t≈ 1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.	
Create equations that describe num-bers or relationships. Extend work on linear and exponential equations in Mathematics I to quadratic equations. Extend A.CED.4 to formulas involving squared variables.	A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s lawV = IR to highlight resistance R.	

Solve equations and inequalities in one variable. Extend to solving any quadratic equation with real coefficients, including those with complex solutions.	A.REI.4 Solve quadratic equations in one variable. a.Use the method of completing the square to transform any qua-dratic equation in x into an equation of the form (x – p)2 = q that has the same solutions. Derive the quadratic formula from this form.b.Solve quadratic equations by inspection (e.g., forx2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Rec-ognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.	
Use complex numbers in polynomial identities and equations.Limit to quadratics with real coefficients.	N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.N.CN.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite x2 + 4 as (x + 2i)(x – 2i).N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	
Solve systems of equations.Include systems consisting of one linear and one quadratic equation. Include systems that lead to work with fractions. For example, finding the intersections between x2 + y2 = 1 and y = (x+1)/2 leads to the point (3/5, 4/5) on the unit circle, corresponding to the Pythagorean triple 32 + 42 = 52	A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = –3x and the circle x2 + y2 = 3	
Unit 4 Applications of Probabilty		
Understand independence and con-ditional probability and use them to interpret data.Build on work with two-way tables from Mathematics I Unit 4 (S.ID.5) to develop understanding of conditional probability and independence.	S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).S.CP.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.S.CP.3 Understand the conditional probability of A given B as P(Aand B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.	

Use the rules of probability to compute probabilities of compound events in a uniform probability model.	S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.S.CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.S.CP.8 (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.S.CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.	
Use probability to evaluate outcomes of decisions.This unit sets the stage for work in Mathematics III, where the ideas of statistical inference are introduced. Evaluating the risks associated with conclusions drawn from sample data (i.e. incomplete information) requires an understanding of probability concepts.	S.MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).S.MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	
Unit 5 Similarity, Right Triangle Trigonometry, and Proof		
Understand similarity in terms of simi-larity transformations.	G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor.a.A dilation takes a line not passing through the center of the dila-tion to a parallel line, and leaves a line passing through the center unchanged.b.The dilation of a line segment is longer or shorter in the ratio given by the scale factor.G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	
Prove geometric theorems.Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Implementation of G.CO.10 may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for G.C.3 in Unit 6.	G.CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.G.CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.G.CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.	
Prove theorems involving similarity.	G.SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	
Use coordinates to prove simple geo-metric theorems algebraically.	G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	

Define trigonometric ratios and solve problems involving right triangles.	G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems	
Prove and apply trigonometric identi-ties.In this course, limit θ to angles between 0 and 90 degrees. Connect with the Pythagorean theorem and the distance formula. A course with a greater focus on trigonometry could include the (+) standard F.TF.9: Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. This could continue to be limited to acute angles in Mathematics II. Extension of trigonometric functions to other angles through the unit circle is included in Mathematics III.	F.TF.8 Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin (θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle.	
Unit 6 Circles With an Without Coordinates		
Understand and apply theorems about circles.	G.C.1 Prove that all circles are similar.G.C.2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.G.C.4 (+) Construct a tangent line from a point outside a given circle to the circle.	
Find arc lengths and areas of sectors of circles.Emphasize the similarity of all circles. Note that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course.	G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	
Translate between the geometric de-scription and the equation for a conic section.Connect the equations of circles and parabolas to prior work with quadratic equations. The directrix should be parallel to a coordinate axis	G.GPE.1 Derive the equation of a circleof given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.G.GPE.2 Derive the equation of a parabola given a focus and directrix.	
Use coordinates to prove simple geo-metric theorems algebraically.Include simple proofs involving circles.	G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).	
Explain volume formulas and use them to solve problems.Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k2 times the area of the first. Similarly, volumes of solid figures scale by k3 under a similarity transformation with scale factor k.	G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri’s principle, and informal limit arguments.G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	